Structure preserving model reduction of large-scale second-order linear dynamical systems

*** Tsuyoshi Koyama[1] and Sanjay Govindjee[2]**

[1] ETH Zurich, IMES,
CLA H23.1, Tannenstr.3,
CH-8092 Zurich, Switzerland
tkoyama@ethz.ch

[2] ETH Zurich, IMES,
CLA J27, Tannenstr.3,
CH-8092 Zurich, Switzerland
gsanjay@ethz.ch

Second-order linear dynamical systems appear in many physical systems of interests such as electrical, mechanical, electromechanically coupled, and thermomechanically coupled systems. When continuous systems of these types are discretized by a numerical method they lead to a large-scale system of equations which one must solve to obtain either the transient or steady-state behavior. In many cases one is interested in specific behavior, for example the response output of a scalar quantity of the system with respect to a scalar input quantity. This system can be characterized by the time-invariant single-input single-output second-order system of equations,

$$\mathbf{M}\ddot{\mathbf{z}}(t) + \mathbf{D}\dot{\mathbf{z}}(t) + \mathbf{K}\mathbf{z}(t) = \mathbf{b}u(t) \; , \tag{1}$$

$$y(t) = \mathbf{l}^* \mathbf{z}(t) \; , \tag{2}$$

where $\mathbf{M}$, $\mathbf{D}$, and $\mathbf{K}$ are the mass, damping, and stiffness matrices, $\mathbf{z}$ is the vector of state variables, $\mathbf{b}$ is the vector representing the forcing (input) pattern, $\mathbf{l}$ is the vector representing the sensing (output) pattern, $u(t)$ is the time-varying scalar input, and $y(t)$ is the time-varying scalar output of interest. For steady-state response, one can further take the Laplace transform of the equations to obtain the transfer function in second-order form,

$$H(s) = \mathbf{l}^* \left(s^2\mathbf{M} + s\mathbf{D} + \mathbf{K} \right)^{-1} \mathbf{b} \; . \tag{3}$$

The solution of Eqn. (1) or the evaluation of Eqn. (3) can be computationally expensive when the size of the matrices $\mathbf{M}$, $\mathbf{D}$, and $\mathbf{K}$ are on the order of hundreds of thousand and larger. Ideally one would like to construct a reduced order model which represents the motion of the dynamical system accurately in a specific regime of inputs.

For nonlinear dynamical systems, the method of principal orthogonal decomposition (POD) or principal component analysis where a reduced model is constructed from data of the motion of the full dynamical system in combination with balanced realization has been used successfully [1,2]. For linear dynamical systems, Krylov subspace techniques which take advantage of the linearity of the system have been shown to be a powerful technique with applications in large-scale simulations of circuits and structural dynamics [3]. Krylov subspace techniques have the ability of matching the moments of the reduced system transfer function with those of the full system transfer function. The moments of the transfer function $H(s)$ are defined as the coefficients of its power series expansion around a given point. Since the second-order dynamical system of Eqn. (1) is linear, an approach based on Krylov subspace model reduction can be taken.

Krylov subspace techniques for reduced order modeling are based on constructing a Krylov subspace for the linear operators of the dynamical system, onto which the original state space variables are projected. The size of the subspace is orders of magnitudes smaller than the full system reducing computational effort. In the original development of these techniques, higher-order dynamical systems are converted to first-order form through the introduction of auxiliary variables, from which linear operators are extracted. Theorems have been developed to prove moment matching of the transfer functions based on this first-order form. Such a conversion can have disadvantages such as ignoring the physical meaning of the original matrices, structure of the matrices, and a lack in preservation of the second-order form. This can lead to loss of stability and passivity in the transfer function. Methods treating the second-order system in its original form have been proposed in the context of structure preservation and second-order Krylov subspaces [4,5].

In our work, the moment matching theorems for transfer functions proven through projectors for the first-order form in [5] are proven for the second-order form involving second-order Krylov subspaces. Given this theorem, moment-matching properties of transfer functions for second-order systems are proven without conversion to first-order form. Besides enabling easier treatment of second-order systems, it enables one to prove advantages of incorporating structure preservation in the subspace construction for model reduction. An example for such a case is presented through the model reduction of the equations for linearized thermoelasticity.

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