ETH
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HiQLab: Simulation of Resonant MEMS

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IMES
Institut für
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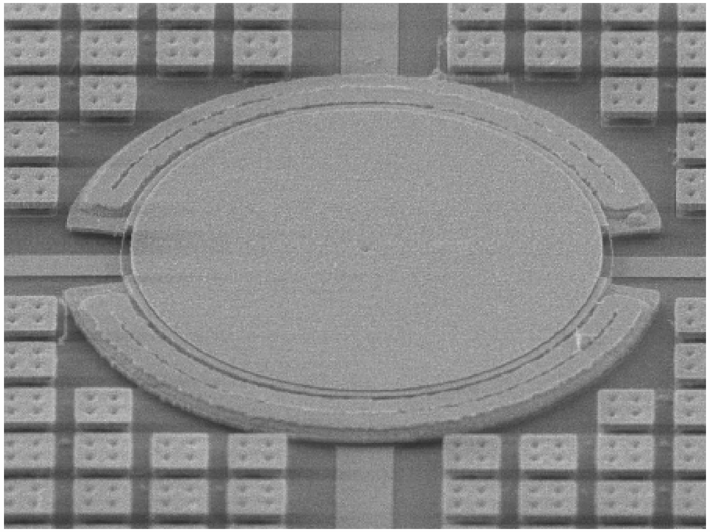
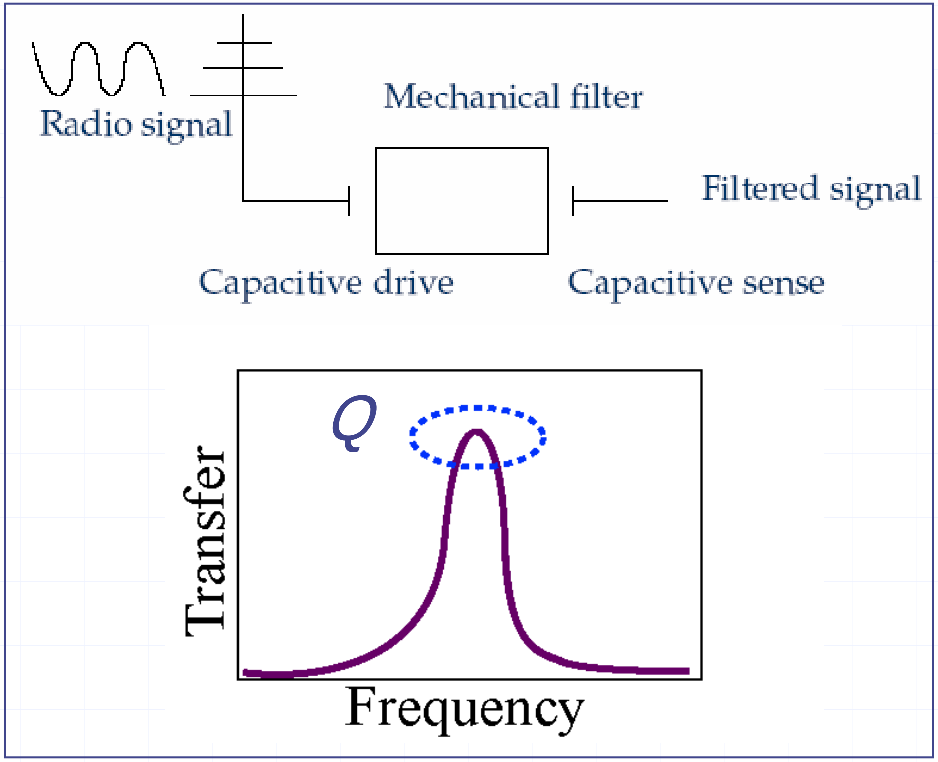
High-frequency MEMS resonators(MHz-GHz)

- Applications as small size, low energy consuming frequency references, filters, and sensors

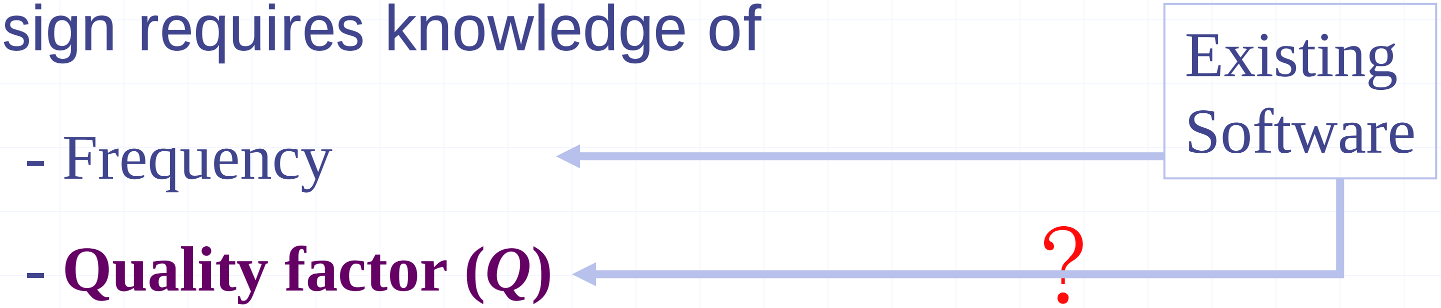


SEM of 41.5 um radius poly-SiGe disk resonator



Resonator Simulation

- Design requires knowledge of



Existing Software

- Frequency
- **Quality factor (Q)**

?

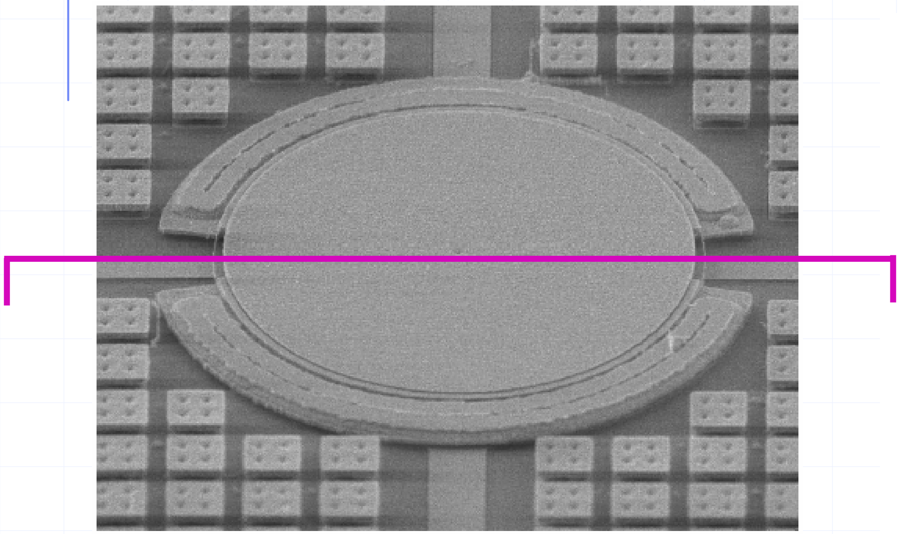
$$Q = \frac{\text{Maximum Stored Energy}}{\text{Energy Loss per radian}} \approx \frac{1}{\text{Damping}}$$

HiQLab:Tool for evaluating damping in resonant MEMS

- Finite element method (FEM) based tool for coupled problems
- Physical damping models (Thermoelastic damping, Anchor loss)
- Efficient algorithms (eigenfrequency computation, Arnoldi based reduced order models)
- Matlab and Lua interface

Disk resonator (Anchor loss)

- Mechanism: Energy loss from radiating waves escaping into the substrate.



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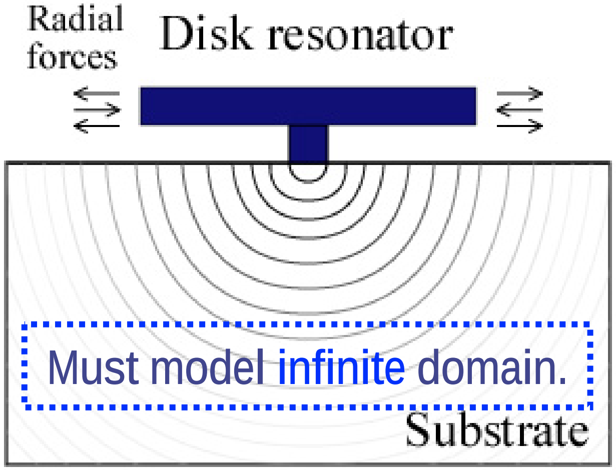
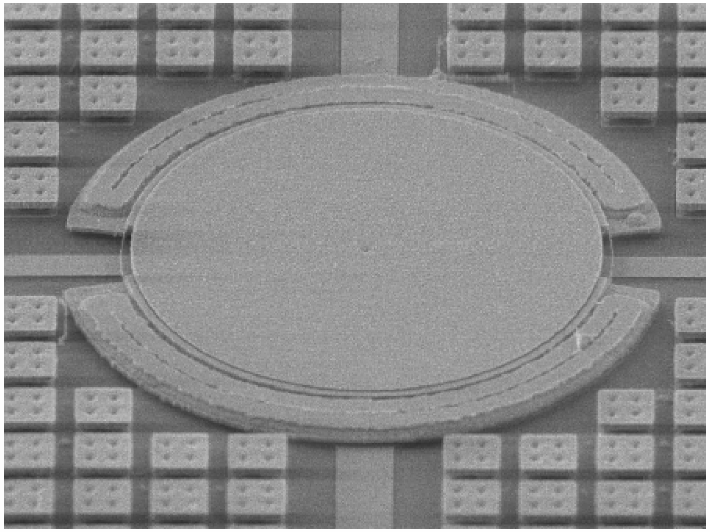


Must model infinite domain.

Section of disk resonator

Perfectly Matched Layers (PML)

- Mechanism: Energy loss from radiating waves escaping into the substrate.



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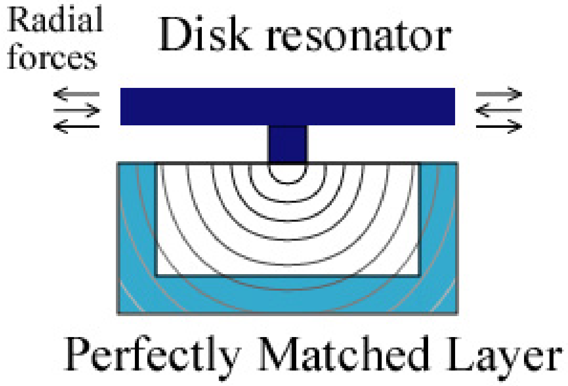
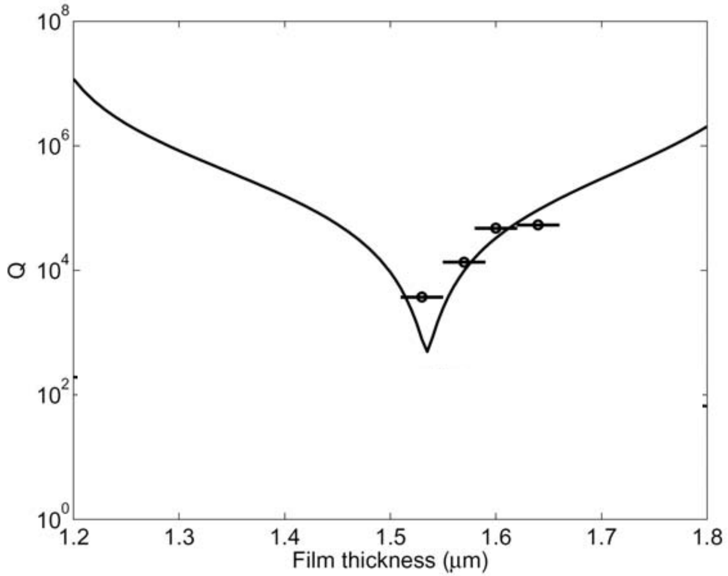


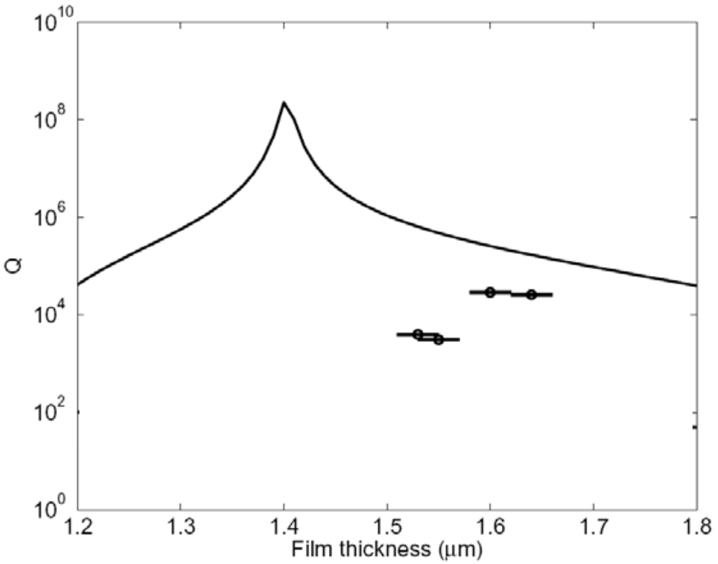
Outgoing waves are absorbed with zero impedance mismatch at PML boundaries.

Uniprocessor axisymmetric results

- Measured Q with varying disk thickness



40um radius disks



30um radius disks

- Investigate 3D effects

- PML requires fine mesh for convergence

Parallel Computing!

Frequency and Q

- Equation of motion discretized with FEM under harmonic assumption

Quadratic eigenvalue problem.

$$\left(-\omega^2 M + i\omega C + K\right)x = 0$$

Complex eigenfrequency

$$\omega = \omega_R + i\omega_I$$

$$\boxed{Q = \frac{|\omega|}{2\omega_I}}$$

Implementation in parallel computing

Sparse eigenvalue problem

- PML implementation results in:

Complex symmetric
generalized eigenvalue problem

$$Kx = \omega^2 M$$

Difficulty: Complex valued, Non-Hermitian

- Krylov subspace based method

1. Strategy to control projection subspace of eigenvalue problem

\+

2. Apply operator to expand subspace at each step. (Linear system solver)

Parallel implementation

◆ Petsc (C)

- Complex valued
- Parallel iterative solvers
- Parallel preconditioners
- Interface to parallel direct solvers

◆ Trilinos (C++)

- Real valued
- Parallel iterative solvers
- Parallel preconditioners
- Interface to parallel direct solvers

Complex valued implementations

1. Slepc (Krylov-Schur)
2. Basis Petsc iterative solvers and preconditioners

Complex vector objects

1. Anasazi (Block Krylov-Schur)
2. SuperLU_Dist (complex)

Results

- Petsc (C)
  - Fails for Axisymetric, 2.8k DOF, 1 process
- Trilinos (C++)
  - Solves 3D, 120k DOF, 4 process using 12GB
  - Fails 3D, 407k DOF, 64 process using 113GB, 30min.

Observations

Both cases fail in the linear solver for expansion of subspace

- Iterative fails without good complex valued preconditioner
- Direct methods take large amounts of memory and may not be able to factorize and in adequate time.

Alternative solutions for linear solve

- Direct method:
  - Complex valued solver MUMPS
  - Reduce fill in with more efficient nodal ordering algorithm
- Iterative method:
  - Preconditioners for complex-valued matrices (Some exist for application to PML)
  - Solve with the equivalent real formulation and use pre-existing rich software

HiQLab group and Software

- PIs: Prof. Sanjay Govindjee (ETHZ)
  Prof. James Demmel (CS and Math, Berkeley)
  Prof. Roger Howe (Stanford)
- Post doctoral students:
  Dr. David Bindel (Courant Institute, NYU)
  Dr. Emmanuel Quevy(Electrical Eng., Berkeley)
- Graduate students:
  Wei He (Civil Engineering, Berkeley)
  Members of the SUGAR group
- HiQLab: Resonant MEMS Simulator
  - http://www.cims.nyu.edu/~dbindel/hiqlab